Homework Feedback 10

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**P. 176 #7** Use the following data and the knowledge that the first five derivatives of *f* are bounded on [1,5] by 2, 3, 6, 12 and 23, respectively, to approximate as accurately as possible. Find a bound for the error.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 | 5 |
| f(x) | 2.4142 | 2.6734 | 2.8974 | 3.0976 | 3.2804 |

**Answer:** We need to check the error bound using 1-4 order polynomial approximation of the function f(x) (five points, at most 4th oder polynomial):

Since the error bound of numerical differentiation is , we can compute for each polynomial order

1. 1st order:
2. 2nd order:
3. 3rd order:
4. 4th order:

Therefore, the most accurate approximation of can be obtained through 4th order polynomial. Its error bound is around 0.76.

**P. 177 #3** Let , Use Eq. (4.9) and the values of at x = 0.25,0.5, and 0.75 to

approximate . Compare this result to the exact value and to the approximation found in

Exercise 11 of Section 3.4. Explain why this method is particularly accurate for this problem,

**Answer:** According to Eq.4.9, we have:

Compare to the exact value 0, it is accurate.

Explanation: First, the curve of in the interval (0.25,0.75) is not complicated, it can be approximated very well by second order polynomial. Second, we select two symmetric points 0.25 and 0.75 for this computation.

**P. 195 #7** The Trapezoidal rule applied to gives the value 4, and Simpson's rule gives the value 2. What is ?

**Answer:** According to Trapezoidal rule, we have: . Similarly, with Simpon’s rule, we have: . Therefore,

**P. 195 #9** Find the degree of precision of the quadrature formula:

**Answer:** Justcheck to what degree of polynomial its integration at interval (-1,1) can be precisely computed using this formula.

We can verify that this formula can compute precisely the integration . So, the degree of precision of the formula is 3.

**P. 195 #11** The quadrature formula is exact for all polynomials of degree less than or equal to 2. Determine .

**Answer:** we have thee unknowns. So, let us construct three equations with respect to :

Through Gauss-elimination, we get:

**P. 195 #13** Find the constants and so that the quadrature formula:

has the highest possible degree of precision.

**Answer:** Sincethere are 3 unknowns, we need three equations to determine the 3 unknowns. Thus, we can at least precisely approximate the polynomials to order 2: .

After solver the three equations constructed by the integration of

and .

**P. 226 #1** Approximate the following integrals using Gaussian quadrature with n = 2, and compare your results to the exact values of the integrals.

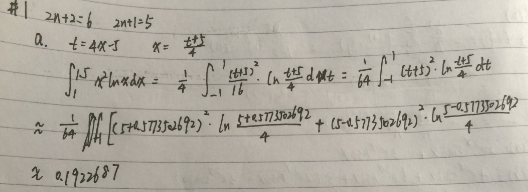
**Answer:** I choose to use Gauss-Legendre polynomial for this problem. According to the textbook, the first 4th polynomials for Legendre polynomials are:

Therefore, the Gaussian quadrature points are -0.7746,0,0.7746, if x is in interval

Let us take (1) as an example:

Typical errors:

Only two Gaussian points are considered, but the problem requires that n=2 gaussian quadrature (*not a serious error, in my point of view, just do not follow the symbol convention in our ppt*):



**P. 226 #5** Determine constants a, b, c, and d that will produce a quadrature formula

that has degree of precision 3.

**Answer:** Similar to **P. 195 #11**,we can construct 4 equations to solve for a,b,c,d. With the simple algebraic derivation, we have: a =1, b=1, c=1/3, d= 1/3.